Diferansiyel Geometri I Quiz Sınavı Cevap Anahtarı

1) $F(x_1,x_2,x_3) = (x_1\cos x_2,\ x_1\sin x_2,\ x_3)$

$$F_*(\bar{v}_p) = \frac{d}{dt} F(p+t\bar{v})\Big|_{t=0}$$

$$p+tv = \left(2+2t,\ \frac{\pi}{2}-t,\ \pi+3t\right)$$

$$F(p+tv) = \left((2+2t)\cos\left(\frac{\pi}{2}-t\right),\ (2+2t)\cdot\sin\left(\frac{\pi}{2}-t\right),\ \pi+3t\right)$$

$$\frac{d}{dt} F(p+t\bar{v}) = \left(2\cos\left(\frac{\pi}{2}-t\right)+(2+2t)\sin\left(\frac{\pi}{2}-t\right),\ 2\cdot\sin\left(\frac{\pi}{2}-t\right)-(2+2t)\cdot\cos\left(\frac{\pi}{2}-t\right),\ 3\right)$$

$$\frac{d}{dt} F(p+tv)\Big|_{t=0} = \left(\underbrace{2\cos\frac{\pi}{2}}_{0}+\underbrace{2\cdot\sin\frac{\pi}{2}}_{1},\ \underbrace{2\sin\frac{\pi}{2}}_{1}-\underbrace{2\cdot\cos\left(\frac{\pi}{2}\right)}_{0},\ 3\right)$$

$$= (2,\ 2,\ 3)_{F(p)} \quad \#$$

2) $\forall\ X = f_1\frac{\partial}{\partial x_1} + f_2\frac{\partial}{\partial x_2} + f_3\frac{\partial}{\partial x_3} \in \mathcal{X}(E^3)$ alalım

$$rot X = \begin{vmatrix} \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ f_1 & f_2 & f_3 \end{vmatrix} = \left(\frac{\partial f_3}{\partial x_2}-\frac{\partial f_2}{\partial x_3}\right)\frac{\partial}{\partial x_1} + \left(\frac{\partial f_1}{\partial x_3}-\frac{\partial f_3}{\partial x_1}\right)\frac{\partial}{\partial x_2} + \left(\frac{\partial f_2}{\partial x_1}-\frac{\partial f_1}{\partial x_2}\right)\frac{\partial}{\partial x_3}$$

$$= \left(\frac{\partial f_3}{\partial x_2}-\frac{\partial f_2}{\partial x_3},\ \frac{\partial f_1}{\partial x_3}-\frac{\partial f_3}{\partial x_1},\ \frac{\partial f_2}{\partial x_1}-\frac{\partial f_1}{\partial x_2}\right)$$

$$div(rot X) = \langle \nabla,\ rot X\rangle$$

$$= \frac{\partial}{\partial x_1}\left(\frac{\partial f_3}{\partial x_2}-\frac{\partial f_2}{\partial x_3}\right) + \frac{\partial}{\partial x_2}\left(\frac{\partial f_1}{\partial x_3}-\frac{\partial f_3}{\partial x_1}\right) + \frac{\partial}{\partial x_3}\left(\frac{\partial f_2}{\partial x_1}-\frac{\partial f_1}{\partial x_2}\right)$$

$$= \cancel{\frac{\partial^2 f_3}{\partial x_1\partial x_2}} - \cancel{\frac{\partial^2 f_2}{\partial x_1\partial x_3}} + \cancel{\frac{\partial^2 f_1}{\partial x_2\partial x_3}} - \cancel{\frac{\partial^2 f_3}{\partial x_2\partial x_1}} + \cancel{\frac{\partial^2 f_2}{\partial x_3\partial x_1}} - \cancel{\frac{\partial^2 f_1}{\partial x_3\partial x_2}}$$

$$= 0 \quad \#$$